Vision Empower & XRCVC

Teacher Instruction KIT

Integers

Syllabus: Karnataka State Board
Subject: Math
Grade: 7
Textbook Name: MATHEMATICS – Text cum Workbook (Revised) – Seventh standard
Chapter Number & Name: 1. Integers

1. OVERVIEW

1.1 OBJECTIVE & PREREQUISITES

Objective

- To understand the properties of usual addition in the set of integers (commutative, associative)
- To understand the multiplication of integers and properties of multiplication of integers (commutative, associative and distributive property)
- To understand division of integers and its properties
- To be able to solve real life situation where the properties are applied

Prerequisite Concept

- Concept: Whole numbers

Chapter Details: TIK_MATH_Grade 6_ Whole numbers

- Concept: Integers

Chapter Details: TIK_MATH_G6_CH6_Integers

Content Index

*Kindly Note: Activities marked with * are mandatory*

2. LEARN

2.1 KEY POINTS

- Integers: An integer is a whole number (not a fractional number) that can be positive, negative, or zero. Examples of integers are: $-5,\ 1,\ 5,\ 8,\ 9,7,$ and $3,043$.

- Definition of a set: A set is a group of things that belong together like the set of even numbers ($2,\ 4,\ 6,\ 8$ and so on)

  Example 1: In a bowl gather a bunch of pebbles. In another bowl gather ten marbles. The first bowl is a set of pebbles. The second bowl is a set of marbles.
  Example 2: In a suitcase there are 5 shirts, 2 trousers, 1 skirt. The suitcase contains a set of clothes. Now I take the 5 shirts and put them in a bag. The bag contains a set of shirts.
  Example 3: The set of integers contains all integers only.
  Example 4: The set of whole numbers contains all whole numbers only.

Addition properties

- Closure property: Let a and b be any integer then $a + b = c$, where c is an integer
- Commutative property: Let a and b be any integer then $a + b = b + a = c$, where c is an integer
- Associative property: The associativeproperty states that you can add or multiply regardless of how the numbers are grouped. Let a, b, c be any integers then

$a + (b + c) = (a + b) + c$

- Additive identity: Let 'a' be an integer then $a + 0 = 0 + a = a$

Multiplicative properties

- Closure property: Let 'a' and 'b' be any integer then
- $a \times b = c$, where c is an integer.
- Commutative property: Let 'a' and 'b' be any integer then $a \times b = b \times a = c$, where c is an integer.
- Associative property: The associativeproperty states that you can add or multiply regardless of how the numbers are grouped. Let a, b, c be any integers then

$a \times (b \times c) = (a \times b) \times c$

- Distributive property: Let a, b, c be any integers then $a \times (b + c) = (a \times b) + (a \times c)$
- Multiplicative identity: Let 'a' be an integer then $a \times 1 = 1 \times a = a$

2.2 LEARN MORE

3. ENGAGE

3.1 INTEREST GENERATION ACTIVITY

Counting steps

Activity 1: Counting steps

Materials Required: None

Prerequisites: Integers

Activity Flow

For this activity, start in the middle of a corridor or passageway. Label the starting point as 0, walk sideways, towards the right and count each step as a positive number, that is 1, 2, 3, ... , n. Come back to the starting point (keep a bag or a stool or some other marker at that spot). Now walk leftwards, counting negative numbers for each step, that is $-1, -2, -3 \ \ldots$

Recall

Activity 2: Recall

Materials Required: Tactile diagram of number line of alphabets representing integers, Number line shows the temperature in degree Celsius ($°C$) at different places on a particular day, tactile diagram of a magic square.

Prerequisites: Number line of integers, addition and subtraction of integers

Activity Flow

Which integers marked by A, G, J, M and O?

Refer: TD_G7_CH1_Integers_Activity 2_Q1

Answer: $A = -7$, $G = -1$, *J = 2, M = 5, O = 7*

Arrange $8, -6, 3,\ 0$ and 11 *in ascending order and then mark them on a number line to check your answer.*

Answer: $-6,\ 0,\ 3,\ 8,\ 11$

State whether the following statements are correct or incorrect. Correct those which are wrong

i. *When two positive integers are added we get a positive integer.*
Answer: True

ii. *When two negative integers are added we get a positive integer.*
Answer: False. When two negative integers are we get a negative integer.

iii. *When a positive integer and a negative integer are added, we always get a negative integer.*
Answer: True

iv. *Additive inverse of an integer 5 is* (-5) *and additive inverse of* (-5) *is 5.*
Answer: True

v. $(-11) + 7 = 11 - 7$
Answer: False. $(-11) + 7 = -11 + 7$
$12 + (-9)(-4) = 12 + 9 - 4$
Answer: False. $12 + (-9) - (-4) = 12 - 9 + 4$

Find a pattern for each of the following.

a. *1, 5, 9, 13, ?, ?. Answer: 17, 21*

b. $-3,\ -6,\ -9,\ -12,\ ?\ ,?$
Answer: $-15, -18$

c. $6,\ 1,\ -4,\ -9,\ ?,\ ?$

Answer: $-14, \ -19$

d. $-11, \ -6, \ -1, \ 4, \ ?, \ ?.$

Answer: 9, 14

Tactile diagram of number line shows the temperature in degree Celsius at different places on a particular day.

Refer: TD_G7_CH1_Integers_Activity2_Q5

a) *Observe the number line and write the temperature of the places marked on it.*
Answer: LahaulSpiti: -8 degree celsius *Srinagar:* - -2 degree celsius, *Shimla:* 5 degree celsius, *Ooty:* 14 degree celsius, *Bangalore:* 22 degree celsius

b) *What is the temperature difference between the hottest and coldest places?*
Answer: Hottest place is Bangalore with 22°C and the coldest place is Lahaul -8 degre celcius. *Then the difference between them is,*

$22-(-8)=22+8=30$ degree celsius

c) *What is the temperature difference between Lahulspiti and Srinagar?*
Answer: Lahaul -8 degree celsius, Srinagar is -2 degree Celsius. *The difference between them is,* $-8-(-2)=-8+2=-6$ degree celsius

d) *Can we say the temperature of Srinagar and Shimla taken together is less than the temperature at Shimla? Is it also less than the temperature at Srinagar?*
Answer: Srinagar is -2 degree celsius, *Shimla is 5 degree celsius. The temperature of Srinagar and Shimla taken together is* $-2+5=3$ *. It is less than the temperature at Shimla.*

Verify $a-(-b)=a+b$ *for the following values of a and b.*

i. *a = 21, b = 18. Answer:* $21\text{-}(-18)=21+18=39$

ii. *a = 118, b = 125. Answer:* $118-(-125)=118+125=243$

iii. *a = 75, b = 84. Answer:* $75-(-84)=75+84=159$

iv. *a = 28, b = 11. Answer:* $28-(-11)=28+11=39$

3.2 CONCEPT INTRODUCTION ACTIVITIES

Properties of Addition and Subtraction of Integers

Closure under Addition

Activity 3: Closure under Addition

Materials Required: None
Prerequisites: Addition of integers (TIK_MATH_G6_CH6_Integers), Addition of whole numbers (TIK_MATH_Grade 6_ Whole numbers)

Activity Flow

Explain to them the following examples then relate it with whole numbers under operation addition. At the end explain the property for integers.
Example 1: set of shirts (Refer Section 1.1 for understanding of sets)

For the operation "wash", the shirt is still a shirt after washing.

- *So shirts are closed under the operation "wash"*

Example 2: set of glasses/vessels

For the operation "wash", the glass is still a glass after washing.

- *So glasses are closed under the operation "wash"*

Example 3:

Similarly, under the operation 'plus ' for the set of whole numbers, we know that addition of the whole number is always a whole number. Example: 17 + 24 = 41. 41 is a whole number. I.e. whole numbers are closed under addition. We know that this property is known as the closure property. Refer: Closure property under activity 3: Properties of whole number in the TIK_MATH_Grade 6_ Whole numbers.
Similarly, integers are closed under addition. In general, for any two integers a and b, a + b is an integer.
For example: $-8+14=6$ *is an integer.*
Similarly, ask the children to check this property for other integers as well.

Closure under Subtraction

Activity 4: Closure under Subtraction
Materials Required: None
Prerequisites: Subtraction of integers

Activity Flow

Under the operation minus, for the set of integers, we find that the result from subtraction of two integers is always an integer.
Example 1: $9-7=2$. *9 and 7 are integers and the result is 2 which is also an integer.*
Example 2: $7-9=-2$. *7 and 9 are integers and the result is* -2 *which is also an integer.*

This shows that integers are closed under subtraction. This property is known as the closure property of subtraction of integers.
In general, for any two integers a and b, $a - b$ *is always an integer.*
Note: This is unlike subtraction of whole numbers which are not closed under subtraction(TIK_MATH_Grade 6_ Whole numbers)

Ask the students to subtract an integer from another integer and observe what number they get and whether that number is an integer or not.

i. $17-(-25)=?$ Answer $17-(-25)=17+25=42$

ii. $-40-21=?$ Answer $=-40-21=-61$

iii. $16-38=?$ Answer $=16-38=-22$

iv. $90-(-75)?$ Answer $=90-(-75)=90+75=165$

v. $(-67)-0=?$ Answer $=(-67)-0=-67$

Commutative property

Activity 5: Commutative property
Materials Required: None
Prerequisites: Addition and subtraction of integers

Activity Flow

The commutative property in math comes from the words "commute" or "move around." This rule states that you can move numbers or variables around and still get the same answer. If the students are not familiar with the concept of commutative property and to apply this property to integers. Refer: Commutative property under activity 3: Properties of whole number in the TIK_MATH_Grade 6_ Whole numbers.

The commutative property is a rule that says that the order in which we add numbers does not change the total.

Then ask the following questions.
Are the following equal?

(i) *12 + (32) and (32) + 12*

(ii) $(-8)+(-9)$ and $(-9)+(-8)$

(iii) $-23+32$ and $32+(-23)$

(iv) $(-45)+0$ and $0+(-45)$

Try this with five other pairs of integers. Ask if the sums are different when the order is changed.
Certainly not. Therefore, we say that addition is commutative for integers.
Therefore in general, for any two integers a and b, we can say that $a + b = b + a$.

Similarly ask the students to check for subtraction, we know that subtraction is not commutative for whole numbers. Ask if it is commutative for integers.
Consider the integers 6 and 5. Is $6-5$ *the same as* $5-6$*? No, because* $6-5=1$*, and* $5-6=-1$

Take at least five different pairs of integers and check this.
Hence, we conclude that subtraction is not commutative for integers.

The property holds for Addition and Multiplication, but not for subtraction and division.

Associative property

Activity 6: Associative property
Materials Required: None
Prerequisites: Addition and Subtraction

Activity Flow

The word associate in associative property means to group or to join or to combine. Therefore, associative property is related to grouping.

Example 1: I went to the supermarket and bought ice cream for 12 rupees, bread for 8 rupees, and milk for 15 rupees.

How much money should I give the cashier?

When I add the prices, I can combine or add the price of the ice cream and the bread first and add the result to the price of milk. That is, (ice cream + bread) + milk

Or, I can combine or add the price of bread and milk first then add the result to the price of ice cream, that is ice cream + (bread + milk). Both ways of approaching the problem gives the same total.

Mathematically, you are trying to do the following:

12 + 8 + 15

You may add these three numbers in the order they appear

$12+8=20$ *(This is adding price of ice cream and bread first)*

You can use parentheses to show the order in which you are adding. Then add the price of milk.

$(12+8)+15$ *which is equivalent to*
$20+15=35$ *(total)*

Another way to add is when you may decide you will add first 8 and 15

$8+15=23$ *(This is adding price of bread and milk first)*

Again, using parentheses to show the order in which you are adding, you get:

$12+(8+15)$ *or*
$12+23=35$ *(total)*

So, we conclude that $(12+8)+15=12+(8+15)$

The above example illustrates the associative property of addition

Terms added in different combinations or grouping yield the same answer

Do it: Teacher may read out items from a shopping bill and ask the child to note down the items and prices in braille and ask groups of children to add the prices of the same items, each group in a different order. They will check if the total price is the same.

Example 2: To bring new basketballs to a sports center, two trucks have arrived with 10 boxes each. Inside each box, there are 8 basketballs. How many basketballs have reached the sports center?

We can answer it by grouping the factors in two ways, but still, end up with the same answer (160). Let's see how this works:

- *If we first multiply the number of trucks by the number of boxes in each truck* (2×10)*, we get the total number of boxes (20). Then, we multiply by the number of basketballs in each box* (20×8) *and that gives us a total of 160 basketballs in all*
- *Now we will solve the problem by grouping the factors in a different way. This time, we will multiply the number of boxes by the number of basketballs in each box first* (10×8) *,which gives us 80, the number of balls in each truck. Then, we multiply by the number of trucks* (80×2) *and we get 160 balls in total*

Observe the following examples: Consider the integers $-3,-2$ and -5 *Look at* $(-5)+[(-3)+(-2)]$ and $[(-5)+(-3)]+(-2)$*. In the first sum* (-3) *and* (-2) *are grouped together and in the second* (-5) *and* (-3) *are grouped together. We will check whether we get different results*. $(-5)+[(-3)+(-2)]$ and $[(-5)+(-3)]+(-2)$

Take five more such examples. You will not find any example for which the sums are different. Addition is associative for integers. In general for any integers a, b and c, we can say
$a + (b + c) = (a + b) + c$

Associative property song:

When adding numbers three or more,
Group it anyway, you will still score!
Don't let adding in order be the aim,
Because the sum will just be the same!

Additive identity

Activity 7: Additive identity
Materials Required: None
Prerequisites: Addition and Subtraction

Activity Flow

Additive identity is a number, which when added to any number, gives the sum as the number itself. It means that the additive identity is "0" as adding 0 to any number, gives the sum as the number itself.

Example: There are 5 kids in a team. No other kid joined the team till the end of the game. How many kids were in the game?

As no one joined the game, this means there was no change in the number of kids.

5 kids + 0 kids = 5 kids

The additive identity property is also called the zero property of addition.

When we add zero to any whole number, we get the same whole number. Zero is an additive identity for whole numbers. Is it an additive identity again for integers also? Observe the following and fill in the blanks:

(i) $(-8)+0=-8$

(ii) $0+(-8)=-8$

(iii) $0+?=-43$

(iv) $-61+?=-61$

(v) $?+10=10$

The above examples show that zero is an additive identity for integers. You can verify it by adding zero to any other five integers. In general, for any integer a, $a + 0 = a = 0 + a$

Multiplication of Integers

Activity 8: Multiplication of a positive and a Negative Integer

Materials Required: Tactile number line of integers

Prerequisites: Multiplication of whole numbers

Activity Flow

Standard numbers, anything greater than zero, are described as 'positive' numbers. We don't put a plus sign (+) in front of them because we don't need to since the general understanding is that numbers without a sign are positive.

Numbers that are less than zero are known as 'negative' numbers. These have a minus sign (−) in front of them to indicate that they are less than zero (for example, -10 *or 'minus 10').*

Let's do an activity

It is all about direction.

- *Ask the students to take the tactile number line of integers and ask them to go back to zero, and face in the negative numbers (direction), that is towards* $-1,\ -2,$ *etc.*
- *Take two steps forwards, then another two. Ask them where they are standing? They are now standing on* -4.

- *So, they have moved twice in the negative direction with two steps. That is .,* $2\times(-2)$ steps $=-4$ *steps. Hence, negative × positive = negative*

Observe the following pattern:

We have,

$3\times-5=-15$

$2\times(-5)=-10$

$1\times(-5)=-5$

We thus find that while multiplying a positive integer and a negative integer, we multiply them as whole numbers and put a minus sign (−) before the product. We thus get a negative integer.

In general, for any two positive integers a and b we can say $a\times(-b)=(-a)\times b=-(a\times b)$

Find:

(i) $6\times(-19)$

Answer: -114

(ii) $12\times(-32)$

Answer: -384

(iii) $7\times(-22)$

Answer: -154

Activity 9: Multiplication of two Negative Integers

Materials Required: Tactile number line of integers

Prerequisites: Multiplication of whole numbers

Activity Flow

Explain the example given below

When I say "Eat!" I am encouraging you to eat (positive)

But when I say "Do not eat!" I am saying the opposite (negative).

Now if I say "Do not eat!", I am saying I don't want you to starve, so I am back to saying "Eat!" (positive).

Let’s do an activity

Ask the students to take the tactile number line of integers and ask them start at zero, face in the negative direction.

Now take two steps backwards, and then another two backwards.

They are now standing on +4.

By facing in the negative direction, and walking backwards (two negatives), they reached a positive number. Hence negative × negative = positive

Observe the pattern of numbers

$(-3)\times(-1)=3=3\times1$

$(-3)\times(-2)=6=3\times2$

$(-3)\times(-3)=9=3\times3$

$(-4)\times(-1)=4=4\times1$

So, $(-4)\times(-2)=4\times2=?$

$(-4)\times(-3)=?=?$

So observing these products we can say that the product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product. Thus, we have $(-10)\times(-12)=+120=120$

Similarly $(-15)\times(-6)=+90=90$

In general, for any two positive integers a and b, $(-a)\times(-b)=a\times b$

Find ,

$(-31)\times(-100)$,

$(-25)\times(-72)$,

$(-83)\times(-28)$

Explain the following example to the students

The enemy $(-)$ of $(*)$ my enemy $(-)$ is my friend $(+)$

The friend $(+)$ of $(*)$ my enemy $(-)$ is my enemy $(-)$

The enemy $(-)$ of $(*)$ my friend $(+)$ is my enemy $(-)$

The friend $(+)$ of $(*)$ my friend $(+)$ is my friend $(+)$

Activity 10: Product of three or more negative integers

Materials Required: None

Prerequisites: Multiplication of integers

Activity Flow

Let us observe the following examples:

$(-4)\times(-3)=12$

$(-4)\times(-3)\times(-2)=[(-4)\times(-3)]\times(-2)=12\times(-2)=-24$

$$(-4)\times(-3)\times(-2)\times(-1)=\left[(-4)\times(-3)\times(-2)\right]\times(-1)=(-24)\times(-1)$$

$$(-5)\times\left[(-4)\times(-3)\times(-2)\times(-1)\right]=(-5)\times 24=-120$$

From the above products we observe that
(a)The product of two negative integers is a positive integer;
(b) The product of three negative integers is a negative integer.
(c) Product of four negative integers is a positive integer.

The product $(-9)\times(-5)\times(-6)\times(-3)$ is positive
whereas the product $(-9)\times(-5)\times 6\times(-3)$ is negative. Why?

What will be the sign of the product if we multiply together:
(a) 8 negative integers and 3 positive integers?
(b) 5 negative integers and 4 positive integers?

Properties of multiplication of integers

Activity 11: Closure under multiplication
Materials required: None
Prerequisites: Addition and Subtraction

Activity Flow

As they already know about the closure property of addition discussed in the above activity. Closure property of multiplication is same as closure property of addition but here the operation is multiplication. Closure is when an operation (such as "multiplication") on members of a set (such as "integers") always makes member of the same set.

$(-20)\times(-5)=100$ Product is an integer

$(-15)\times 17=-255$ Product is an integer

$(-30)\times 12=?$

$(-15)\times(-23)=?$

$(-14)\times(-13)=?$

$12\times(-30)=?$

What do you observe? Can you find a pair of integers whose product is not an integer? No. This gives us an idea that the product of two integers is again an integer. So we can say that integers are closed under multiplication.
In general, $a\times b$ is an integer, for all integers a and b.

Find the product of five more pairs of integers and verify the above statement.

Activity 12: Commutativity of multiplication
Materials Required: None
Prerequisites: Multiplication of integers

Activity Flow

A good way to remember this is to think of the word commutative. What's another word that is similar? Commute is similar. When I think of commuting, I think of commuting to work. Now, if on my way to work, one of the roads I normally take is blocked off due to construction, I would have to take a detour, a different way to get to work. But, even if I take a different route, I still end up in the same location - at work. It's the same with the commutative property of multiplication; you might have to multiply numbers in a different order to make the problem easier to solve, but your end result - your answer - will still be the same.

$$(-4)\times 3 = -12 = 3\times(-4)$$
$$(-30)\times 12 = ? = 12\times(-30)$$
$$(-15)\times(-10) = (-10)\times(-15) = 150$$
$$(-35)\times(-12) = ? = (-12)\times(-35)$$
$$(-17)\times 0 = ? = (-1)\times(-15)$$

The above examples suggest multiplication is commutative for integers. Write five more such examples and verify. In general, for any two integers a and b, $a\times b=b\times a$

Note: Commutative property only applies to multiplication and addition. However, subtraction and division are not commutative.

Activity 13: Associative property
Materials Required: None
Prerequisites: Multiplication

Activity Flow

To "associate" means to connect or join with something.
According to the associative property of multiplication, the product of three or more numbers remains the same regardless of how the numbers are grouped.

Look at this and complete the products:

$\left[(7)\times(-6)\right]\times 4 = ?\times 4 = ?$

$7\times\left[(-6)\times 4\right] = 7\times ? = ?$

Is $\left[7\times(-6)\right]\times 4 = 7\times\left[(-6)\times 4\right]$?

Does the grouping of integers affect the product of integers? No. In general, for any three integers a, b and c, $(a\times b)\times c = a\times(b\times c)$

Take any five values for a, b and c each and verify this property. Thus, like whole numbers, the product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers

What is the difference between commutative and associative properties of Integers?

In commutative property, the integers can be rearranged in any way and the result will still be the same. In case of associative property, integers can be grouped in any way using parenthesis (by taking the common out) and the result will still be the same.
Commutative Property: $\mathrm{a}\times\mathrm{b}=\mathrm{b}\times\mathrm{a}$
Associative Property: $(\mathrm{a}\times\mathrm{b})\times\mathrm{c}=\mathrm{a}\times(\mathrm{b}\times\mathrm{c})$

Activity 14: Multiplication by zero
Materials Required: None
Prerequisites: Multiplication

Activity Flow

We know that any whole number when multiplied by zero gives zero. Observe the following products of negative integers and zero. These are obtained from the patterns done earlier.

$(-3)\times 0 = 0$

$0\times(-4) = 0$

$-5\times 0 = ?$

$0\times(-6) = ?$

This shows that the product of a negative integer and zero is zero

Activity 15: Multiplicative Identity
Materials Required: None
Prerequisites: Multiplication of integers with 1

Activity Flow

According to the multiplicative identity property of 1, any number multiplied by 1, gives the same result as the number itself.
It is also called the Identity property of multiplication, because the identity of the number remains the same.

Check that 1 is the multiplicative identity for integers as well. Observe the following products of integers with 1.

$(-3)\times 1 = -3$

$1\times 5 = 5$

$(-4)\times 1 = ?$

$1\times 8 = ?$

$1\times(-5) = ?$

$3\times 1 = ?$

$1\times(-6) = ?$

$7\times 1 = ?$

This shows that 1 is the multiplicative identity for integers also. In general, for any integer a we have, $a\times 1 = 1\times a = a$

Let's sing
Multiplying numbers by 1 is fun like a game
Because the product will just be the same!

Activity 16: Distributive property
Materials Required: None
Prerequisites: Multiplication and addition of integers

Activity Flow

To "distribute" means to divide something or give a share or part of something.
According to the distributive property, multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Example:

There are 5 rows for girls and 8 rows for boys in the class. If each row has 12 students. Determine the total number of students in the class.

Answer:

Method 1:

Total number of rows = $5+8=13$

Number of students in each row = 12

Total number of students in the class = $13\times12=156$

Method 2:

$12\times(5+8)=12\times13=156$ *or*

$12\times(5+8)=(12\times5)+(12\times8)=60+96=156$

Hence in the above example, we are using the distributive property to simplify the expression in method 2.

Ask the students to observe the following pattern

(a) $(-2)\times(3+5)=-2\times8=-16$

and $[(-2)\times3]+[(-2)\times5]=(-6)+(-10)=-16$

So, $(-2)\times(3+5)=[(-2)\times3]+[(-2)\times5]$

(b) $(-4)\times[(-2)+7]=(-4)\times5=-20$

and $[(-4)\times(-2)]+[(-4)\times7]=8\ +(-28)=-20$

So, $(-4)\times[(-2)+7]=[(-4)\times(-2)]+[(-4)\times7]$

We say that the distributivity of multiplication over addition is true for integers.
In general, for any integers a, b and c, $a\ \times\ (b\ +\ c)\ =\ (a\ \times\ b)\ +\ (a\ \times\ c)$ *Take at least five different values for each of a, b and c and verify the above Distributive property.*
Find:

1. *Is* $10\times(6-(-2)]=10\times6-10\times(-2)$
2. *Is* $(-15)\times[(-7)-(-1)]=(-15)\times(-7)-(-15)\times(-1)$

Let's sing
To multiply big numbers, break one apart,
Multiply with the addends, right at the start,
Now, add the products, part by part.
The answer is right! You're so smart!

Activity 17: Making multiplication easier

Materials Required: None
Prerequisites: Multiplication

Activity Flow

Consider the following:

(i) *We can find* $(-25)\times 37\times 4$ as $\left[(-25)\times 37\right]\times 4=(-925)\times 4=-3700$

Or, we can do it this way,

$(-25)\times 37\times 4=(-25)\times 4\times 37=\left[(-25)\times 4\right]\times 37=(-100)\times 37=-3700$

Which is the easier way? Obviously the second way is easier because multiplication of (-25) *and 4 gives* -100 *which is easier to multiply with 37. Note that the second way involves commutativity and associativity of integers.*

So, we find that the commutativity, associativity and distributivity of integers help to make our calculations simpler. Let us further see how calculations can be made easier using these properties.

(ii) *Find* 16×12,

16×12 *can be written as* $16\times(10+2)$

$16\times 12=16\times(10+2)=16\times 10+16\times 2=160+32=192$

(iii) $(-23)\times 48=(-23)\times[50-2]=(-23)\times 50-(-23)\times 2=(-1150)-(-\ 46)=-1104$

(iv) $(-35)\times(-98)=(-35)\times\left[(-100)+2\right]=(-35)\times(-100)+(-35)\times 2=3500+(-70)=3430$

(v) $52\times(-8)+(-52)\times 2, (-52)\times 2$ can also be written as $52\times(-2)$

Therefore,

$52\times(-8)+(-52)\times 2=52\times(-8)+52\times(-2)=52\times\left[(-8)+(-2)\right]=52\times\left[(-10)\right]=-520$

Find each of the following products:

(i) $(-18)\times(-10)\times 9$

(ii) $(-20)\times(-2)\times(-5)\times 7$

(iii) $(-1)\times(-5)\times(-4)\times(\ 6)$

Division of integers

Activity 18: Division of a negative integer by a positive integer

Materials Required: None
Prerequisites: Division of whole number

Activity Flow

Meera owes 8,000 on her car loan. Each of her 4 children is willing to pay an equal share of this loan. Using integers, determine how much money each of her children will pay.

Solution: Owing 8,000 can be represented by -8000. *We must divide* -8000, *by 4 in order to solve this problem. However, we need rules for dividing integers in order to continue.*

Rule 1: The quotient of a positive integer and a negative integer is a negative integer.

We can now use Rule 1 to solve the problem above arithmetically $-8000 \div +4 = -2000$ *Each of Meera's four children will pay 2,000 rupees.*

Let's look at some more examples of dividing integers using the above rules.

We observe the following :

$$(-12) \div 2 = (-6)$$
$$(-20) \div 5 = (-4)$$
$$(-32) \div 4 = (-8)$$
$$(-45) \div 5 = (-9)$$

We observe that when we divide a negative integer by a positive integer, we divide them aswhole numbers and then put a minus sign (−) before the quotient.

Activity 19: Division of a positive integer by a negative integer

Materials Required: None

Prerequisites: Division of whole number

Activity Flow

A positive number divided by a negative number is actually negative. So, you can't divide any two numbers and expect to get a positive result. Positive and negative numbers have different behaviors in general.

One way to visualize this on a number line is to interpret the minus sign as movement to the left and the (implied) plus sign as movement to the right.

Let's say we want to divide -4 by 2*, how many jumps of size 2 (or, jumps of size 2 to the right) it takes to get to* -4*.*
It has two jumps of size 2 to get to -4*; thus,* $-4 \div 2 = -2$
Positive numbers divided by negative numbers give negative numbers.

Example:
$-8 \div 2 = -4$
-8 divided into groups of 2 gives a result of -4 groups
We observe that:
$72 \div (-8) = -9$ and $50 \div (-10) = -5$
$72 \div (-9) = -8$ and $50 \div (-5) = -10$
So we can say that when we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (−) before the quotient.
In general, for any two positive integers a and b, $a \div (-b) = (-a) \div b$ *where* $b \neq 0$

Activity 20: Division of a negative integer by a negative integer
Materials Required: None
Prerequisites: Division of whole number

Activity Flow

Let's try dividing -4 by -2*. We can start with the same setup by marking* -2 *and* -4 *on the number line. The* -4 *will be to the left of* -2*.*

And now we figure out how many jumps of size -2 *(or, jumps of size 2 to the left) it takes to get to* -4*. It has two jumps of size* -2
So we get a positive number again. Visually, we see that the arrows for $4 \div 2$ *and* $-4 \div -2$ *are simply flipped across 0; this goes along with the idea that the minus signs of* -4 *and* -2 *cancel out.*

We observe that: dividing negative integer by negative integer will give positive integer

$(-12) \div (-6) = 2$
$(-20) \div (-4) = 5$

$(-32) \div (-8) = 4$

$(-45) \div (-9) = 5$

So, we can say that when we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+). In general, for any two positive integers a and b, $(-a) \div (-b) = (a) \div b$ *where b ≠ 0*

Find:

(a) $(-36) \div (-4)$

(b) $(-201) \div (-3)$

(c) $(-325) \div (-13)$

Activity 21: Properties of division of integers

Materials Required: None

Prerequisites: Division of integers

Activity Flow

- *We know that division is not commutative for whole numbers. Let us check it for integers also.*

Observe that: $(-8) \div (-4) \neq (-4) \div (-8)$.

Is $(-9) \div 3$ the same as $3 \div (-9)$?

Is $(-30) \div (-6)$ the same as $(-6) \div (-30)$?

Can we say that division is commutative for integers? No.

We can verify it by taking five more pairs of integers.

- *Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero that is, for any integer a,* $a \div 0$ *is not defined*

But $0 \div a = 0$ *for* $a \neq 0$.

- *When we divide a whole number by 1 it gives the same whole number. Let us check whether it is true for negative integers also.*

Observe the following:

$(-8) \div 1 = (-8)$

$(-25) \div 1 = ?$

$(-37) \div 1 = ?$

$(-48) \div 1 = ?$

This shows that a negative integer divided by 1 gives the same negative integer. So, any integer divided by 1 gives the same integer.

In general, for any integer a, $a \div 1 = a$

- *What happens when we divide any integer by* (-1) *?*

Complete the following:

$(-8) \div (-1) = 8$

$11 \div (-1) = -11$

$(-25) \div (-1) = ?$

What do we observe?

We can say that if any integer is divided by (-1) it does not give the same integer.

- Can we say $[(-16) \div 4] \div (2)$ is the same as $(-16) \div [4 \div (-2)]$?

    We know that

and $(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

So $[(-16) \div 4] \div (-2) \neq (16) \div [4 \div (-2)]$

Can you say that division is associative for integers? No

Ask the children to verify it by taking five more examples of their own.

Fill in the blanks:

a) $(-31) \div 1 = ?$

b) $(-75) \div ? = -1$

c) $(-206) \div ? = 1$

d) $-87 \div ? = 87$

e) $? \div 1 = -87$

3.3 LET'S DISCUSS: RELATE TO DAILY LIFE*

- Historic timelines (*A.D.*, *B.C.*) are similar to integers, where *B.C.* can be thought of as negative numbers.
- Temperature scales like Centigrade, Fahrenheit use negative numbers to describe temperatures below a certain level
- Banks and credit unions frequently use negative integers. Negative integers can be used to represent debits and positive integers represent credits.
- Sea level is a good example because wherever you go; you are either above or below sea level. An example would be if you were travelling over a mountain, you might be +1372 m above sea level, or if you are in a submarine you could be -57 m below sea level.

4. EXERCISES & REINFORCEMENT

4.1 EXERCISES & REINFORCEMENT

Practice and Recall

Activity 22: Climb up and Down

Materials Required: Two Braille dice of different size, Tactile diagram of Table of numbers from -104 to 104

Pre-requisites: Addition and subtraction of integers

Activity Flow

i. *Take a board marked from* -104 to 104 *as shown in the figure.*
ii. *Number of dots on the bigger dice indicates positive integers and the number of dots on the smaller dice indicates negative integers.*
iii. *Every player will place his/her counter at zero.*
iv. *Each player will take out two dice at a time from the bag and throw them.*
v. *After every throw, the player has to multiply the numbers marked on the dice.*
vi. *If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards* -104
vii. *The player who reaches either* -104 *or 104 first is the winner*

Activity 23: Practice

Materials Required: None
Pre-requisites: Math operations on Integers

Activity Flow

The most effective way of learning this concept is through practice and repetition, so solving a lot of problems is the best concept reinforcement. Make the students solve examples on paper using the Braille typewriter or slate, so that she/he has some reference of (correctly) solved examples to read at a later day (in addition to the text books).
Quiz the student by asking her/him to solve some problems independently.
4.2 IMPORTANT GUIDELINES*

Exercise Reading
It is very important that the children practice their learnings as well as their reading. Hence have the children read out the newly learned concepts from their textbooks or other available resources.

Perform Textbook Activity

It is good practice to have the children perform the textbook activities. Your textbook activities might not be accessible hence go through this resource to learn how to make textbook content accessible

Provide Homework

To evaluate their understanding and to help the student revise and implement the new learnt concept ensure to provide them with homework. Students should perform one or two of the questions mentioned above or from the textbook exercises with the teacher in Class and the remaining may be given for homework. Also, ensure that the student knows their special skills linked to independently using their accessible books as it will be critical to doing homework independently

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